

You have probably noticed that your grapher moves slowly when graphing NINT. This is because it must compute each value as a limit of sums—comparatively slow work even for a microprocessor. Here are some ways to speed up the process:

Change the *x-resolution*. The default resolution is 1, which means that the grapher will compute a function value for every vertical column of pixels. At resolution 2 it computes only every second value, and so on. With higher resolutions, some graph smoothness is sacrificed for speed.

Fundamental Theorem of Calculus

THEOREM 4 The Fundamental Theorem of Calculus, Part 1

If f is continuous on $[a, b]$, then the function

$$F(x) = \int_a^x f(t) \, dt$$

has a derivative at every point x in $[a, b]$, and

$$\frac{dF}{dx} = \frac{d}{dx} \int_a^x f(t) \, dt = f(x).$$

$$\frac{d}{dx} \int_a^x f(t) \, dt = f(x). \quad (1)$$

EXAMPLE 1 Applying the Fundamental Theorem

Find

$$\frac{d}{dx} \int_{-\pi}^x \cos t \, dt \quad \text{and} \quad \frac{d}{dx} \int_0^x \frac{1}{1+t^2} \, dt$$

by using the Fundamental Theorem.

EXAMPLE 2 The Fundamental Theorem with the Chain Rule

Find dy/dx if $y = \int_1^{x^2} \cos t \, dt$.

$$\begin{aligned}\int_1^{x^2} t^3 \, dt &= \frac{1}{4} t^4 \Big|_1^{x^2} \\ &= \frac{1}{4} (x^2)^4 - \frac{1}{4} (1)^4 \\ &= (x^2)^3 (2x) = 2x^7\end{aligned}$$

EXAMPLE 3 Variable Lower Limits of Integration

Find dy/dx .

$$(a) y = \int_x^5 3t \sin t \, dt \qquad (b) y = \int_{2x}^{x^2} \frac{1}{2 + e^t} \, dt$$

Example 1

Given $\frac{dy}{dx} = 3x^2 + 4x - 5$ with the initial condition $y(2) = -1$, find $y(3)$.

Method 1:

Method 2:

EXAMPLE 4 Constructing a Function with a Given Derivative and Value

Find a function $y = f(x)$ with derivative

$$\frac{dy}{dx} = \tan x$$

that satisfies the condition $f(3) = 5$.

$$f(x) = \int_3^x \tan t \, dt + 5$$

EXPLORATION 2 The Effect of Changing a in $\int_a^x f(t) dt$

The first part of the Fundamental Theorem of Calculus asserts that the derivative of $\int_a^x f(t) dt$ is $f(x)$, regardless of the value of a .

1. Graph NDER (NINT $(x^2, x, 0, x)$).
2. Graph NDER (NINT $(x^2, x, 5, x)$).
3. Without graphing, tell what the x -intercept of NINT $(x^2, x, 0, x)$ is. Explain.
4. Without graphing, tell what the x -intercept of NINT $(x^2, x, 5, x)$ is. Explain.
5. How does changing a affect the graph of $y = (d/dx) \int_a^x f(t) dt$?
6. How does changing a affect the graph of $y = \int_a^x f(t) dt$?

THEOREM 4 (continued) The Fundamental Theorem of Calculus, Part 2

If f is continuous at every point of $[a, b]$, and if F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x) \, dx = F(b) - F(a).$$

This part of the Fundamental Theorem is also called the **Integral Evaluation Theorem**.

EXAMPLE 5 Evaluating an Integral

Evaluate $\int_{-1}^3 (x^3 + 1) dx$ using an antiderivative.

$$\begin{aligned} &= \left. \frac{x^4}{4} + x \right|_{-1}^3 \\ &= \left(\frac{3^4}{4} + 3 \right) - \left(\frac{(-1)^4}{4} + (-1) \right) \\ &= 24 \end{aligned}$$

Example 5

A pizza with a temperature of 95°C is put into a 25°C room when $t = 0$. The pizza's temperature is decreasing at a rate of $r(t) = 6e^{-0.1t}^{\circ}\text{C}$ per minute. Estimate the pizza's temperature when $t = 5$ minutes.

initial condition

$$T(x) = 95 - \int_0^x 6e^{-0.1t}$$

$$\begin{aligned} T(5) &= 95 - \int_0^5 6e^{-0.1t} dt \\ &= 71.392^{\circ} \end{aligned}$$

Total Area vs Net Area

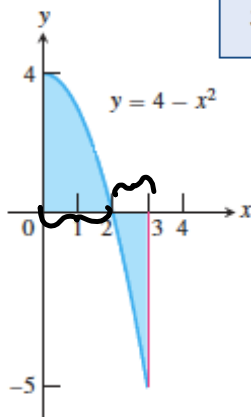
EXAMPLE 6 Finding Area Using Antiderivatives

Find the area of the region between the curve $y = 4 - x^2$, $0 \leq x \leq 3$, and the x -axis.

How to Find Total Area Analytically

To find the area between the graph of $y = f(x)$ and the x -axis over the interval $[a, b]$ analytically,

1. partition $[a, b]$ with the zeros of f ,
2. integrate f over each subinterval,
3. add the absolute values of the integrals.



calculator

total area:

$$\int_0^3 |4 - x^2| dx$$

net area:

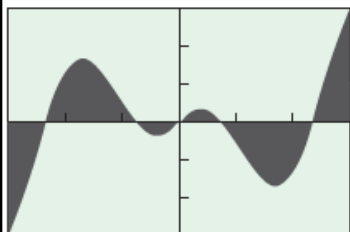
$$\int_0^3 4 - x^2 dx$$

Figure 5.28 The function $f(x) = 4 - x^2$ changes sign only at $x = 2$ on the interval $[0, 3]$. (Example 6)

by hand (must separate positives and neg)

$$\int_0^2 4 - x^2 dx + \left| \int_2^3 4 - x^2 dx \right|$$

We can find area numerically by using NINT to integrate the *absolute value* of the function over the given interval. There is no need to partition. By taking absolute values, we automatically reflect the negative portions of the graph across the x -axis to count all area as positive (Figure 5.29).



$[-3, 3]$ by $[-3, 3]$

(a)



$[-3, 3]$ by $[-3, 3]$

(b)

Figure 5.29 The graphs of (a) $y = x \cos 2x$ and (b) $y = |x \cos 2x|$ over $[-3, 3]$. The shaded regions have the same area.

EXAMPLE 7 Finding Area Using NINT

Find the area of the region between the curve $y = x \cos 2x$ and the x -axis over the interval $-3 \leq x \leq 3$ (Figure 5.29).

How to Find Total Area Numerically

To find the area between the graph of $y = f(x)$ and the x -axis over the interval $[a, b]$ numerically, evaluate

$$\text{NINT}(|f(x)|, x, a, b).$$

EXAMPLE 8 Using the Graph of f to Analyze $h(x) = \int_a^x f(t) dt$

The graph of a continuous function f with domain $[0, 8]$ is shown in Figure 5.30. Let h be the function defined by $h(x) = \int_1^x f(t) dt$.

(a) Find $h(1)$. 0

(b) Is $h(0)$ positive or negative? Justify your answer.

neg: area is above x-axis
and going from right to left

(c) Find the value of x for which $h(x)$ is a maximum.

$f(4)$

(d) Find the value of x for which $h(x)$ is a minimum.

$f(8) \rightarrow$ smaller area
than $f(0)$

(e) Find the x -coordinates of all points of inflections of the graph of $y = h(x)$.

$1, 3, 6$

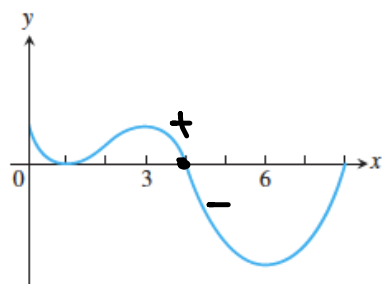


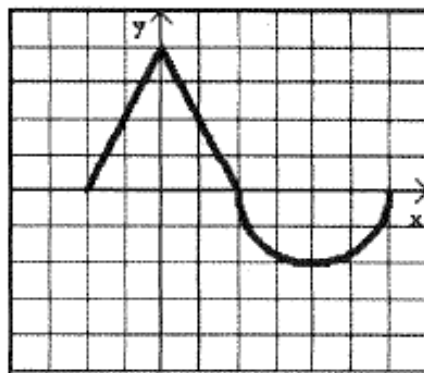
Figure 5.30 The graph of f in Example 8, in which questions are asked about the function $h(x) = \int_1^x f(t) dt$.

Example 3

The graph of f' on $-2 \leq x \leq 6$ consists of two line segments and a semicircle as shown at right.

Given that $f(-2) = 5$,

find $f(0)$, $f(2)$, and $f(6)$.



Graph of f'